Graphs of inverse trig relations

When we graph the inverse sine relation, we plot the values of the sine function (i.e., the points y in the interval $[-1,1]$) on the horizontal axis, and on the vertical axis we plot (the measures of) all the angles θ such that $\sin\theta = y$. Since the sine function is periodic, there are infinitely many points of the graph of the inverse sine relation with the same value of the sine function.

Now we'll construct the graph of the inverse sine relation, and we'll do it in stages. At each stage, we'll use a different color, to make it easy to see the part that's being added.

First, we'll graph (in red) the part of the graph that corresponds to the angles θ in the (vertical) interval $[0,2\pi)$.

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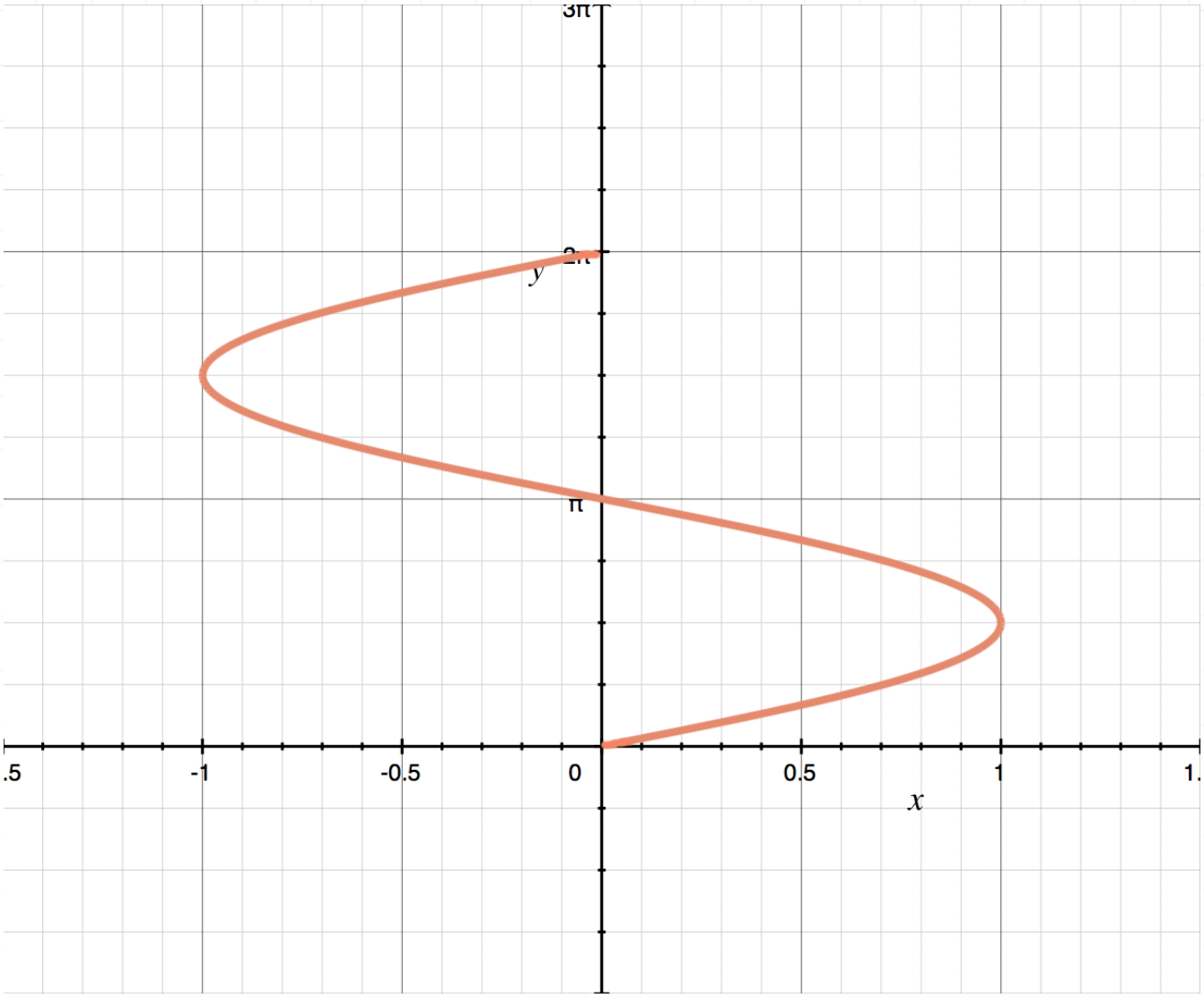

Notice that as the value of the sine function increases from 0 to 1, the angle θ increases from 0 to $\pi/2$. Then as the value of the sine function decreases from 1 to 0 and on down to -1, the angle θ increases from $\pi/2$ to π and on up to $3\pi/2$. Finally, as the value of the sine function increases from -1 to 0, the angle θ increases from $3\pi/2$ to 2π.

Next, we'll add (in light blue) the part that corresponds to the angles θ in the (vertical) interval $[-2\pi,0)$.

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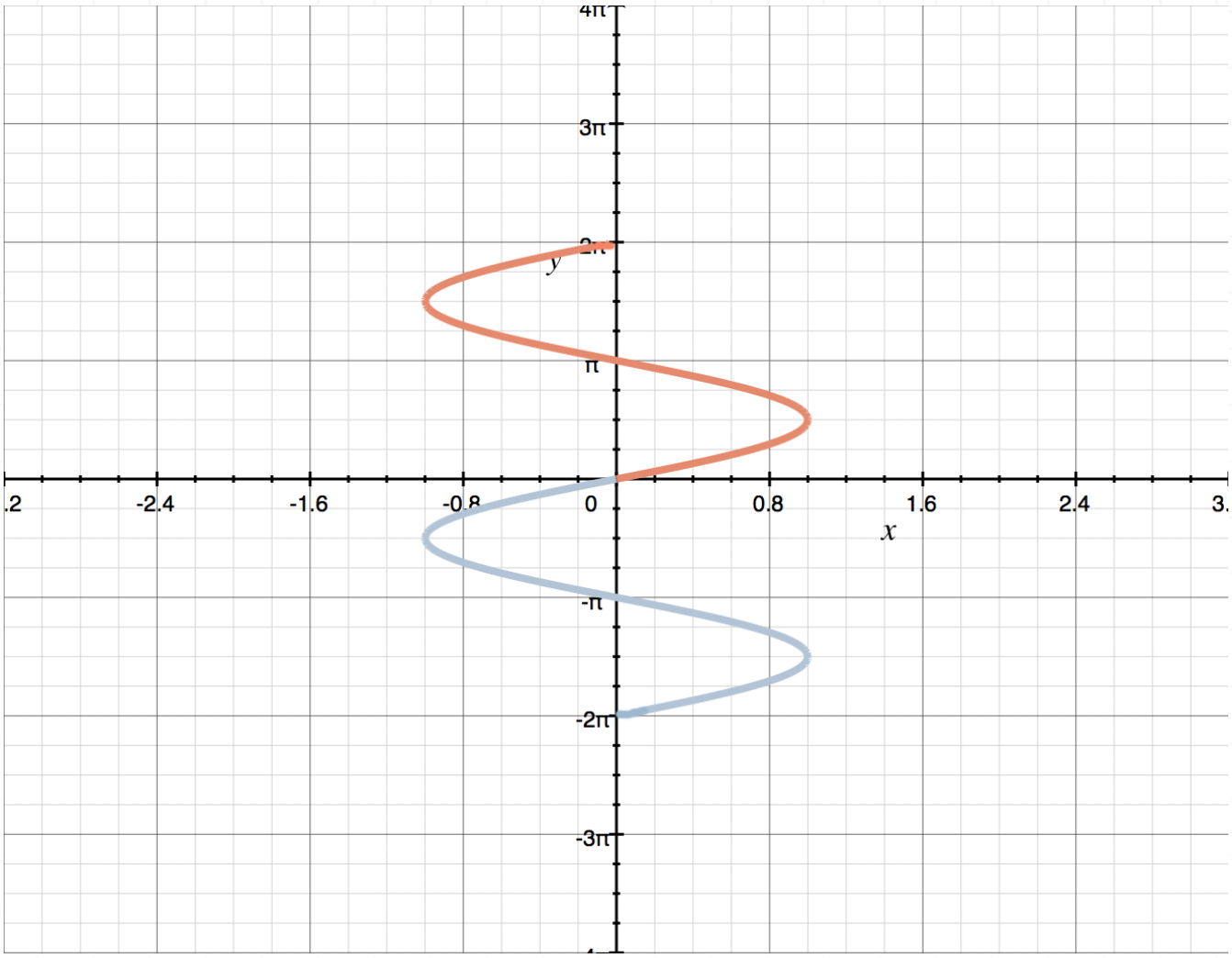

Now we'll add (in dark blue) the parts that correspond to the angles θ in the (vertical) interval $[-4\pi, -2\pi)$ and the (vertical) interval $[2\pi,4\pi)$.

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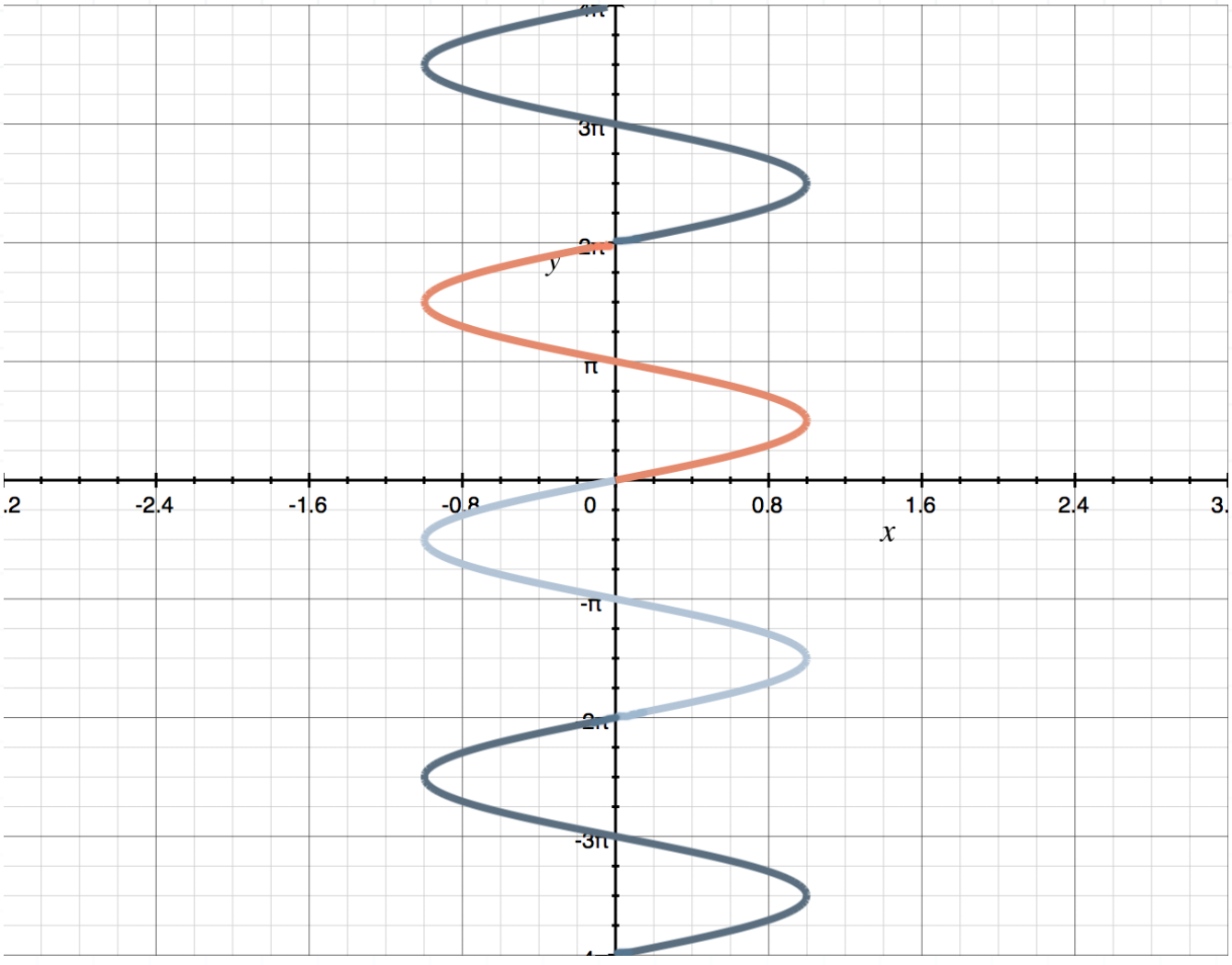

Finally, we'll add (in purple) the parts that correspond to the angles θ in the (vertical) interval $[-6\pi, -4\pi)$ and the (vertical) interval $[4\pi,6\pi)$.

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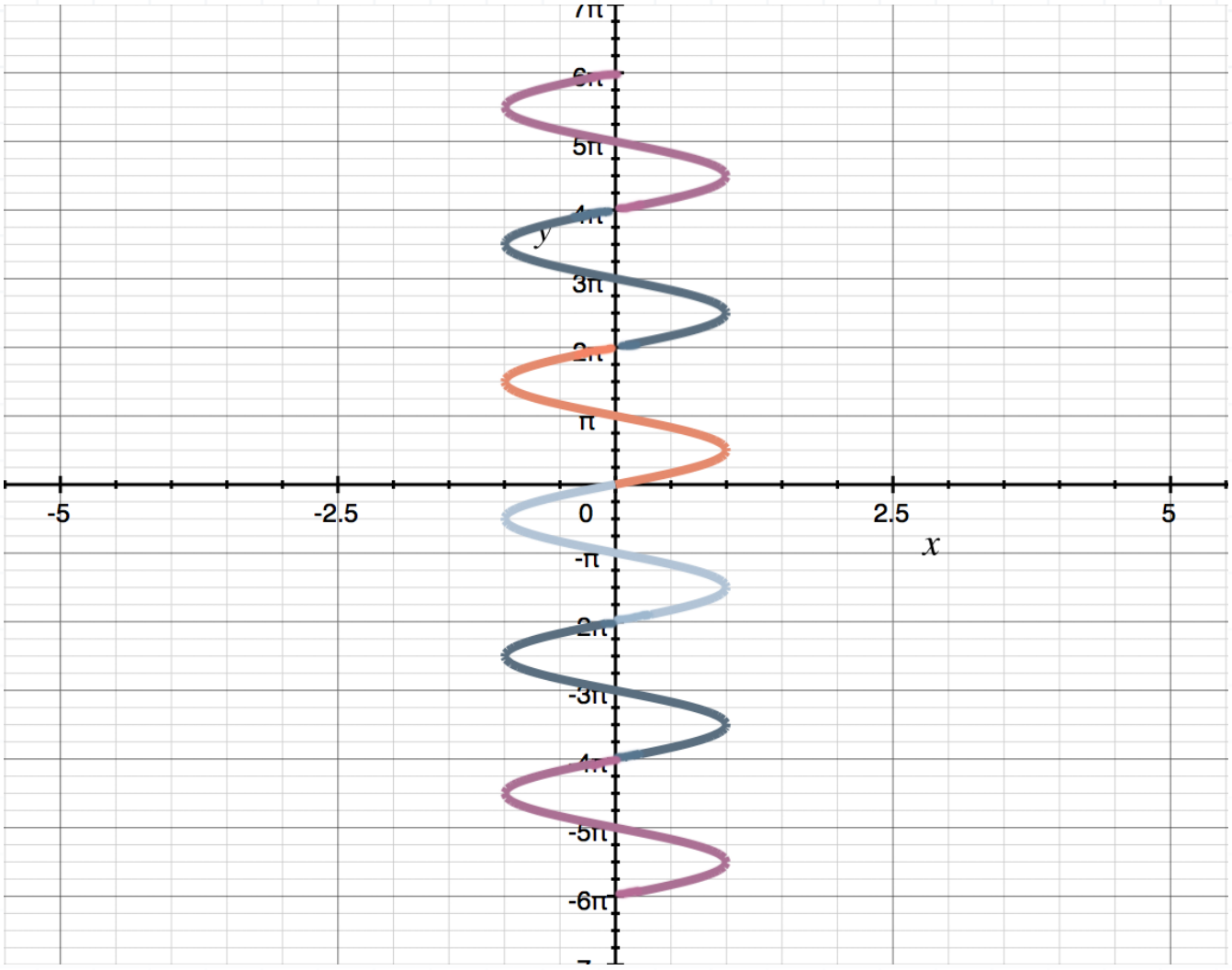

One way to describe the graph of the inverse sine relation is to say that the size and shape of the graph are identical on consecutive vertical intervals (of angle measure) of height 2π, and that 2π is the smallest height for which this is true.

In order to graph the entire inverse sine relation, of course, we would have to keep on going with this process, so we would have infinitely many vertical intervals (of angle measure) of height 2π above the horizontal axis, and infinitely many vertical intervals of height 2π below the horizontal axis.

The same kinds of things happen when we graph the inverse cosine relation, the only difference being the particular infinite set of angles θ that corresponds to any given value of the cosine function.

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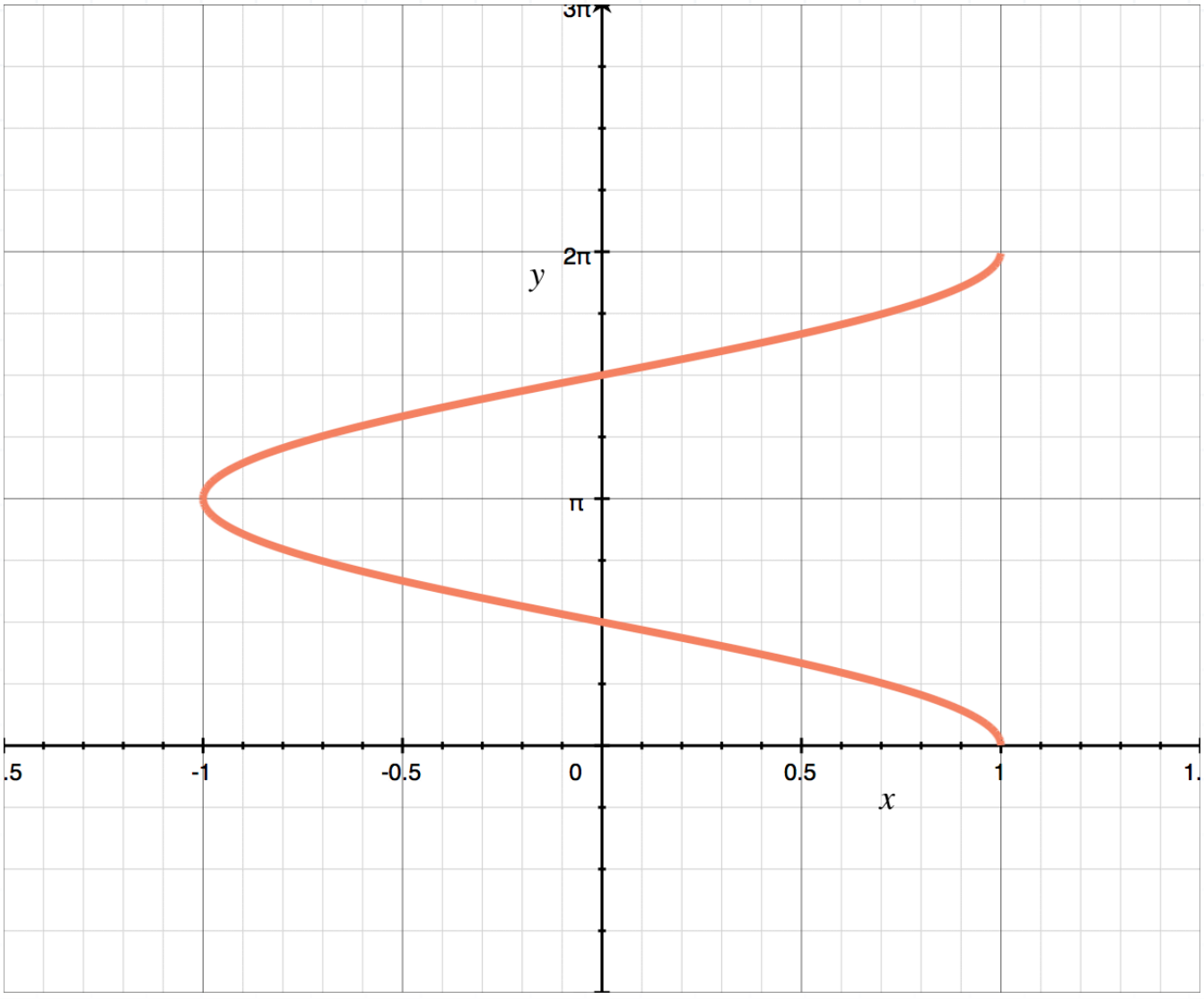

Notice that as the value of the cosine function decreases from 1 to 0 and on down to -1, the angle θ increases from 0 to $\pi/2$ and on up to π. Then as the value of the cosine function increases from -1 to 0 and on up to 1, the angle θ increases from π to $3\pi/2$ and on up to 2π.

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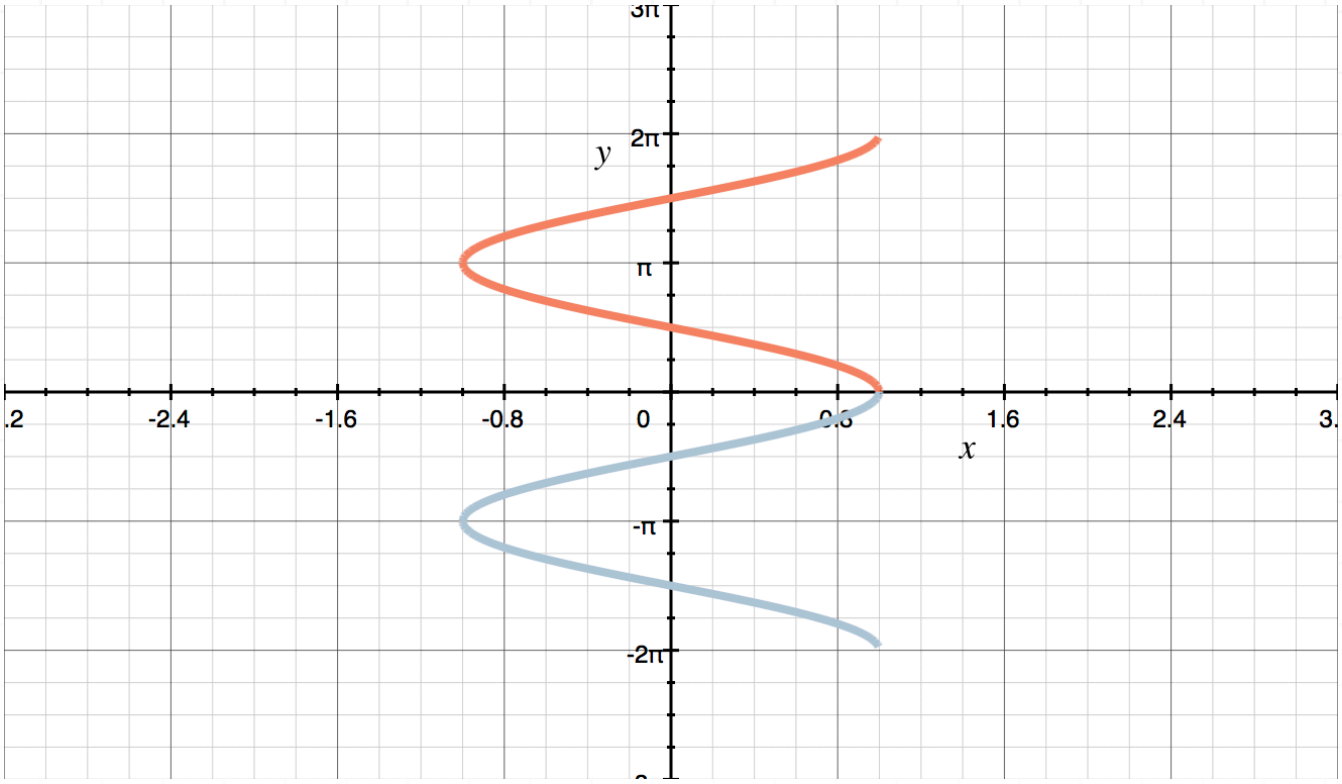

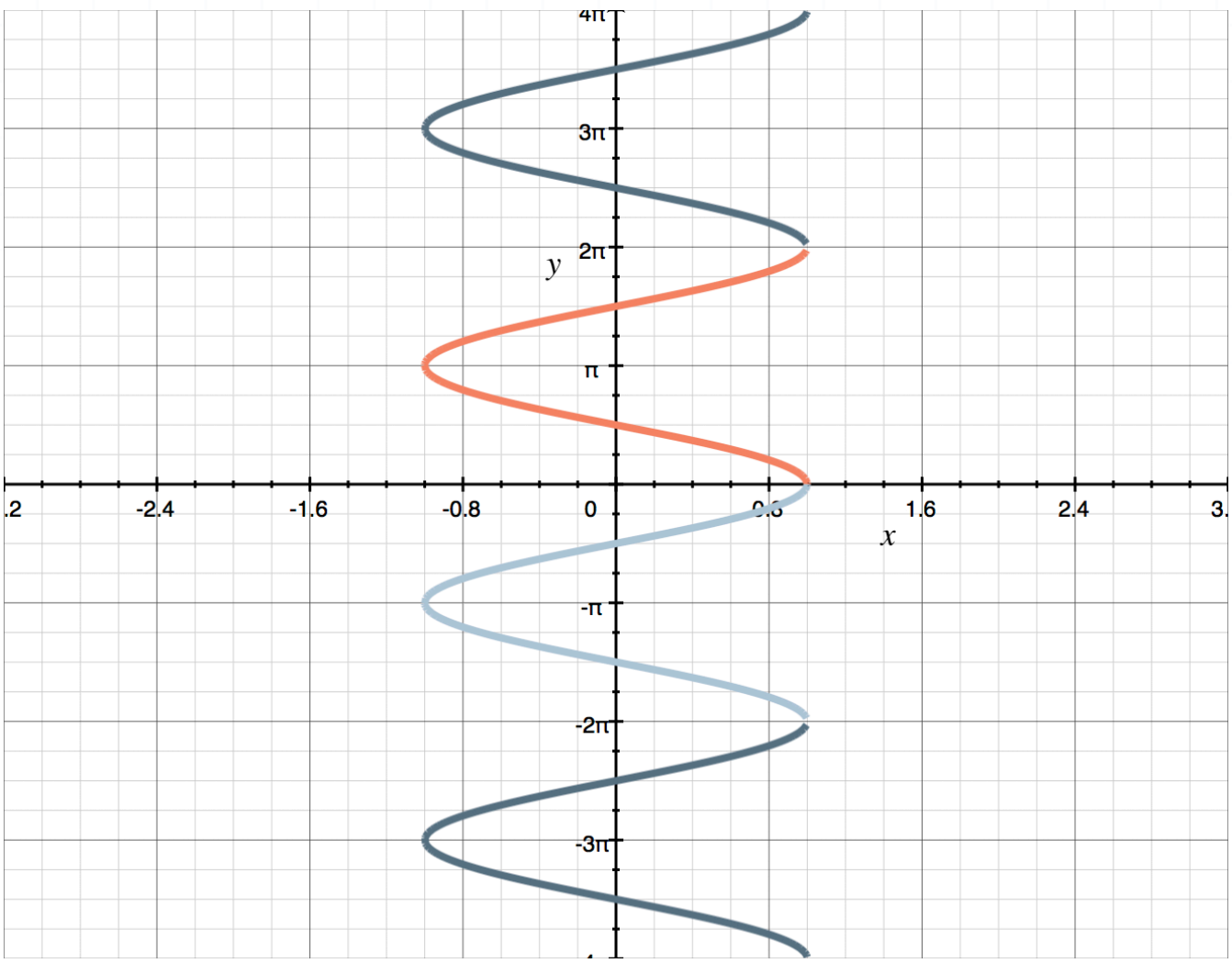

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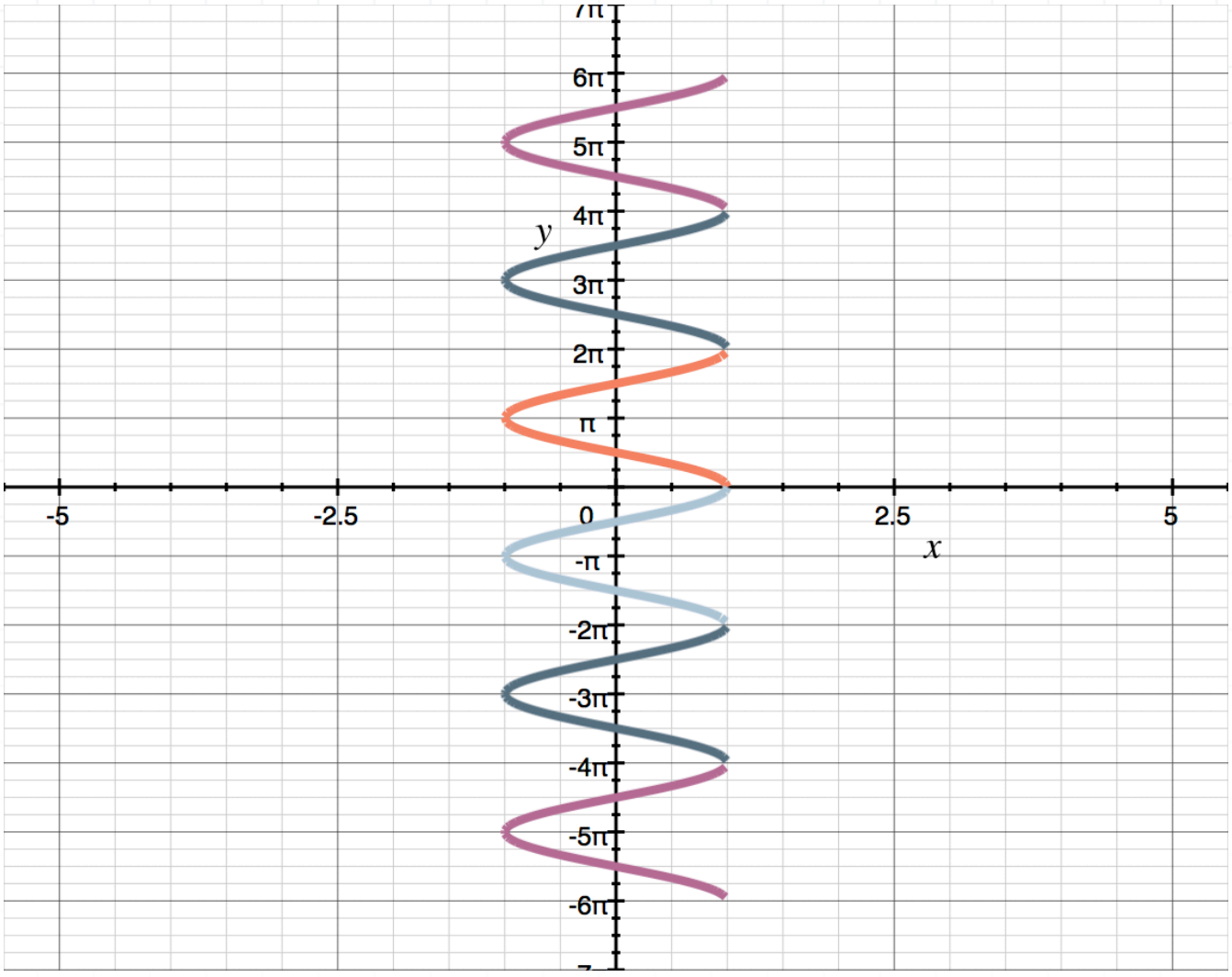

If you wanted to graph the inverse relation for a sine or cosine function other than the basic sine or cosine function (e.g., for a sine or cosine function that has a different period or amplitude than the basic sine or cosine function, or for a sine or cosine function whose graph involves a horizontal or vertical shift), you would have to adjust the graph of the inverse relation accordingly.

Example

Sketch the graph of the inverse relation for the function $\cos(\theta/2)$.

The period of the function $\cos(\theta/2)$ is $2\pi/(1/2) = 4\pi$, so the smallest height for which the size and shape of the graph of the inverse relation for this

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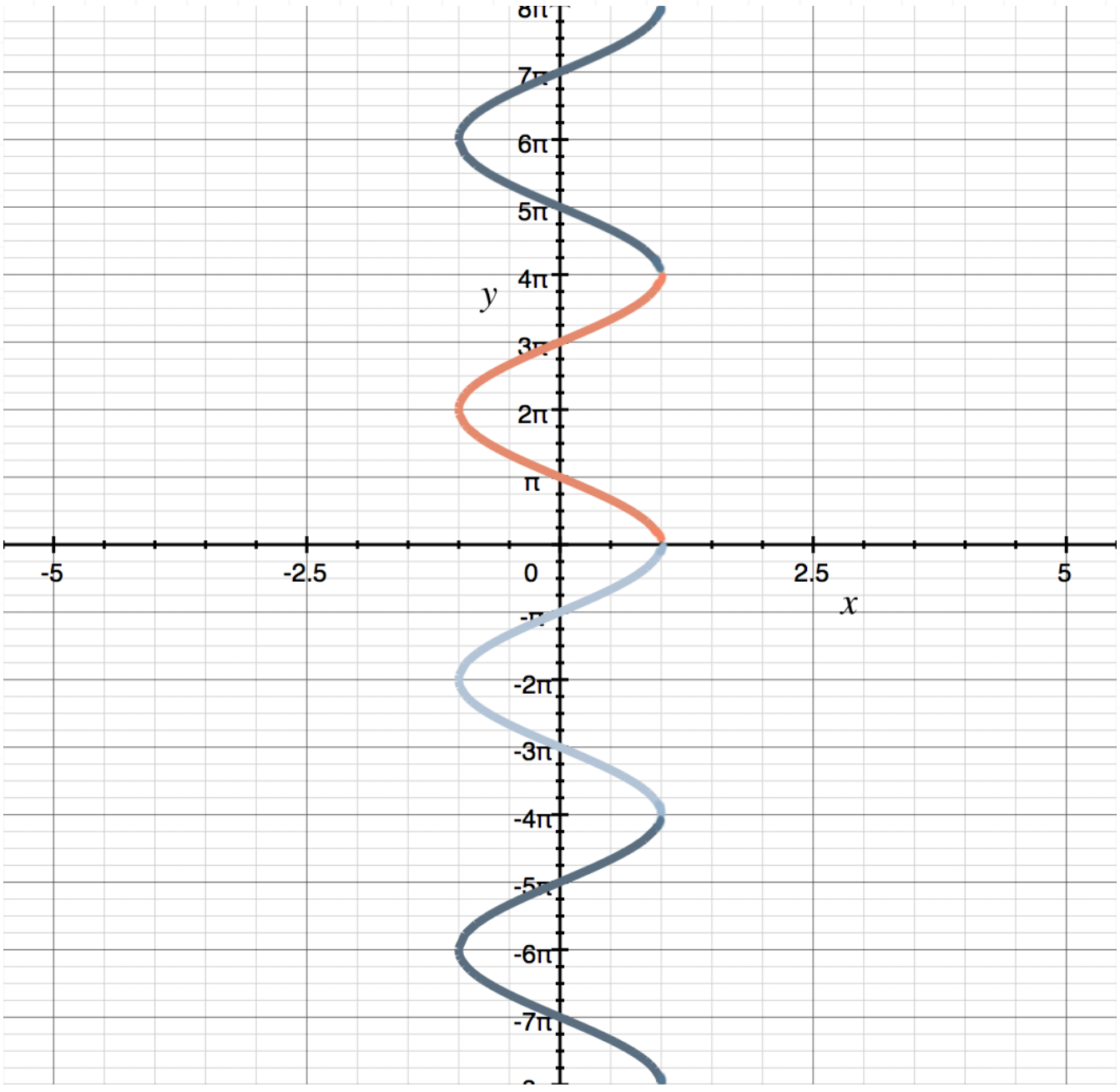
function are identical on consecutive vertical intervals (of angle measure) is 4π (not 2π, as is the case for the basic cosine function).



Let’s look at another example.

Example

Sketch the graph of the inverse relation for the function $\sin(\theta - (\pi/3))$.

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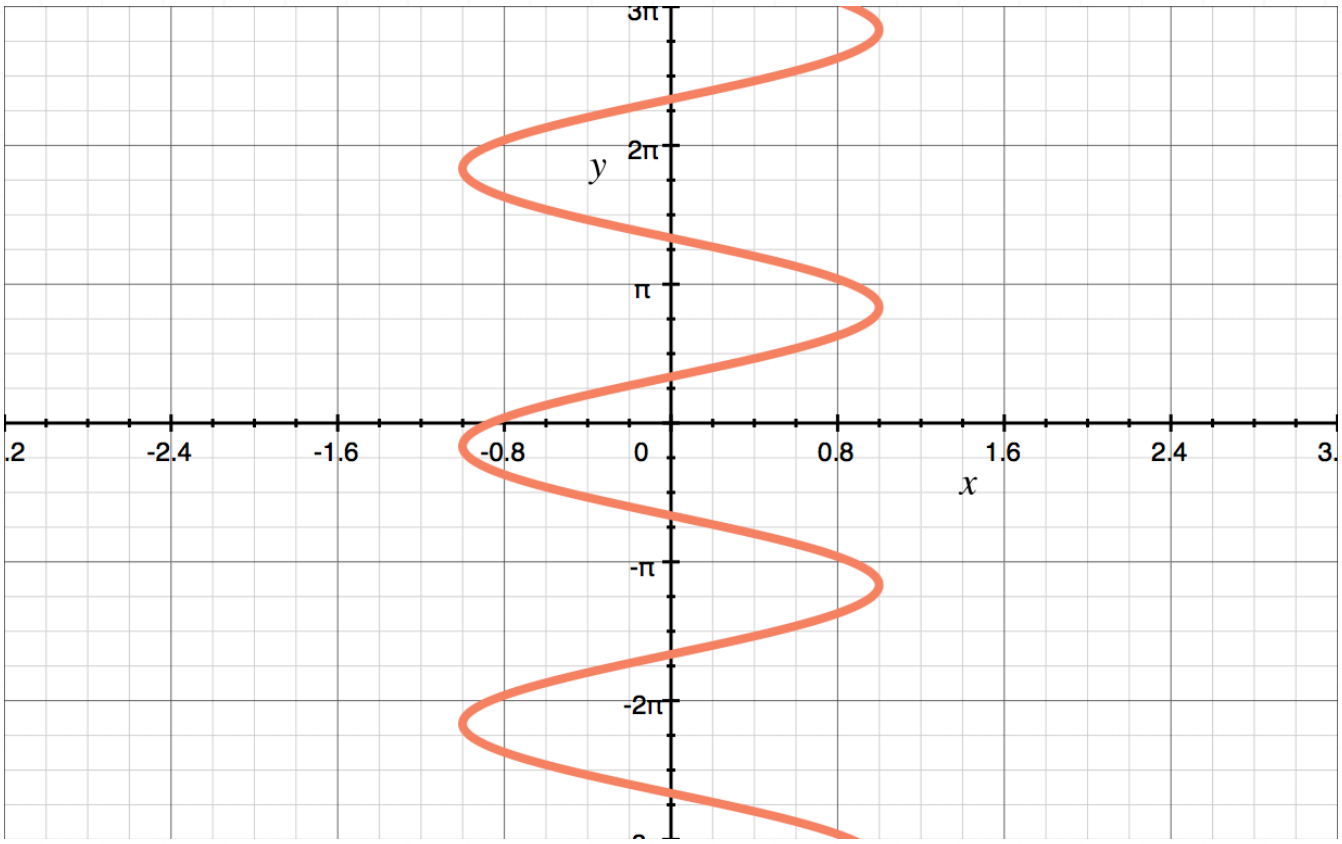
As you may recall, the graph of the function $\sin(\theta - (\pi/3))$ is horizontally shifted by $\pi/3$ units to the right (relative to the graph of $\sin\theta$). Thus the graph of the inverse relation for this function is shifted upwards by $\pi/3$ units.



If you aren't convinced of this, it may help to set

$$\sin\left(\theta - \frac{\pi}{3}\right)$$

to 0 and solve for θ.

We know that the sine of an angle is equal to 0 if and only there is some integer n such that (the measure of) that angle is equal to $n\pi$. Thus

$$\sin\left(\theta - \frac{\pi}{3}\right) = 0$$

if and only if there is some integer n such that

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$$\theta - \frac{\pi}{3} = n\pi$$

Solving for θ, we get

$$\theta = n\pi + \frac{\pi}{3}$$

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